Further Evidence of a Smooth Phase in 4D Simplicial Quantum Gravity *

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Four-dimensional (4D) simplicial quantum gravity coupled to U(1) gauge fields has been studied using Monte-Carlo simulations. A negative string susceptibility exponent is observed beyond the phase-transition point, even if the number of vector fields (N_V) is 1. We find a scaling relation of the boundary volume distributions in this new phase. This scaling relation suggests a fractal structure similar to that of 2D quantum gravity. Furthermore, evidence of a branched polymer-like structure is suggested far into the weak-coupling region, even for $N_V > 1$. As a result, we propose new phase structures and discuss the possibility of taking the continuum limit in a certain region between the crumpled and branched polymer phases.

1. Introduction

The development of simplicial quantum gravity started with the 2D case. Recently, the phase structure for 4D pure simplicial quantum gravity has been intensely investigated as a first step. In 4D pure gravity, two distinct phases are known. For small values of the bare gravitational coupling constant the phase is the so-called elongated phase, which has the characteristics of a branched polymer. For large values of the bare gravitational coupling constant the phase is the so-called crumpled phase. Numerically, the phase transition between the two phases has been shown to be 1st order. As a result, it is difficult to construct a continuum theory. Our second step is to investigate an extended model of 4D quantum gravity. From calculations in ref.[1] we have tried introducing vector fields. Actually, we have treated pure gravity coupled to U(1) gauge fields and have considered the possibility of taking a continuum limit. In order to investigate the phase structures, we mainly measured the string susceptibility exponent (γ_{st}) using the Minbu method [2] and the boundary volume distribution [3]. The aim of this article is to discuss the new phase (smooth phase) in 4D simplicial quantum gravity coupled to gauge fields.

2. Models with Gauge Fields

We start with the Euclidean Einstein-Hilbert action in 4D for pure gravity:

$$S_{EH}[\Lambda, G] = \int d^4x \sqrt{g}(\Lambda - \frac{1}{G}R), \tag{1}$$

where Λ is the cosmological constant and G is Newton's constant. We use discretize action for pure gravity, $S_P[\kappa_2, \kappa_4] = \kappa_4 N_4 - \kappa_2 N_2$, where $\kappa_2 \sim \frac{1}{G}$, κ_4 is related to Λ and N_i is the number of i-simplexes. We use the plaquette action for

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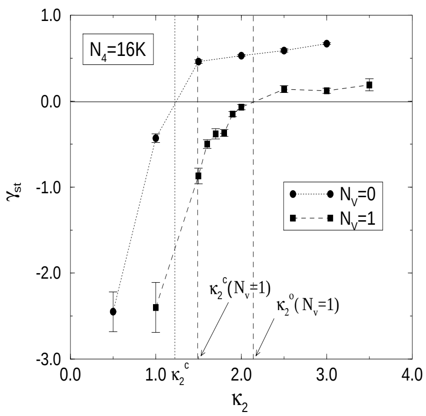

Figure 1. String susceptibility exponents (γ_{st}) plotted versus the coupling constant (κ_2) for $N_v = 0$ and 1.

U(1) gauge fields [4],

$$S_G = \sum_{t_{ijk}} o(t_{ijk})[A(L_{ij}) + A(L_{jk}) + A(L_{ki})]^2, \quad (2)$$

where L_{ij} denotes a link with vertices i and j, t_{ijk} denotes a triangle with vertices i, j and k, $A(L_{ij})$ denotes the $U(1)$ gauge field on a link L_{ij} and $o(t_{ijk})$ denotes the number of simplexes sharing triangle t_{ijk}. The total action of pure gravity with $U(1)$ gauge fields is $S = S_P + S_G$. We consider a partition function, $Z(\kappa_2, \kappa_4) = \sum_T W(T) \int \prod_{L \in T} dA(L) e^{-S_P - S_G}$, where $W(T)$ is the symmetry factor. We sum over all 4D simplicial triangulations (T) in order to carry out a path integral about the metric. Here, we fix the topology with S^4.

3. Numerical Results

In this section we report on two numerical observations: the γ_{st} and the boundary volume distributions. The γ_{st} is defined by the asymptotic form of the partition function, $Z(V_4) \sim V_4^{\gamma_{st}-3} e^{\mu V_4}$, where V_4 denotes the 4D volume. In Fig.1 we plot γ_{st} for various numbers of gauge fields versus κ_2 with volume $N_4 = 16K$. What is important in the $N_V = 1$ case is that the usual phase-transition point (κ_2^c) is different from another transition point (κ_2^o) which separates the $\gamma_{st} < 0$ region from the $\gamma_{st} > 0$ region and γ_{st} becomes negative at the phase-transition point κ_2^c. This fact leads to the definition of a new smooth phase. The new smooth phase is defined by an intermediate region between these two transition points, κ_2^c and κ_2^o. In the pure-gravity case it is clear that $\kappa_2^c \approx \kappa_2^o$, and thus there is no evidence for the existence of a new smooth phase. On the other hand, in the case of $N_V = 1$ with $N_4 = 16K$, we observe the $\gamma_{st} < 0$ region beyond the usual phase-transition point (κ_2^c). We also observe a very obscure transition from $\gamma_{st} < 0$ to $\gamma_{st} > 0$ at κ_2^o (see in Fig.1). This obscure transition is very similar to that of $c = 1$ in 2D quantum gravity. In 2D the $c = 1$ barrier is well known as an obscure transition from the fractal phase ($c \leq 1$ and $\gamma < 0$) to the branched polymer phase ($c > 1$ and $\gamma > 0$). In the $N_V = 1$ case we observe a smooth phase which is separated from the crumpled phase by κ_2^c, and observe the branched polymer phase which is separated from the smooth phase by κ_2^o. In order to investigate statistical structures of these three phases we have observed the boundary volume distributions (ρ) in 4D Euclidian space-time using the concept of geodesic distances. In order to discuss the universality of the scaling relations, we assume that $D^{-\alpha} \cdot \rho(V, D)$ is a function of a scaling variable, $x = V/D^{\alpha}$ [3]. Here, V denotes the volume of the boundary and D is the geodesic distance. This assumption has been justified in 2D [5]. The scaling parameter $\alpha = d_f - 1$ is also defined in the same manner as in ref.[3]. Here, d_f denotes the fractal dimension. In Fig.2 we plot the boundary volume distributions for various geodesic distances for $N_V = 1$ with $N_4 = 16K$ in the smooth phase ($\kappa_2 = 1.7$). The distributions at different distances show excellent agreement with each other. It is clear that the 4D simplicial manifold becomes fractal in the sense that sections of the manifold at different distances from a given 4-simplex look exactly the same after a proper rescaling of the boundary volume. Furthermore, the shape of this scaling function is very similar to that of the 2D case [5,6]. The best account for this excellent agreement in the 4D case can be found in the dominance of a conformal mode and a fractal property. It seems reasonable to suppose that this new smooth phase has a similar fractal structure to that of the 2D fractal surface, and

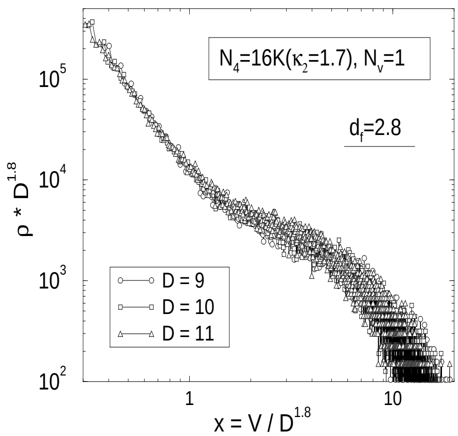

Figure 2. Boundary volume distributions plotted versus the scaling variable (x) with $N_4 = 16K$ (in the smooth phase: $\kappa_2 = 1.7$) for $N_V = 1$ using $log - log$ scales.

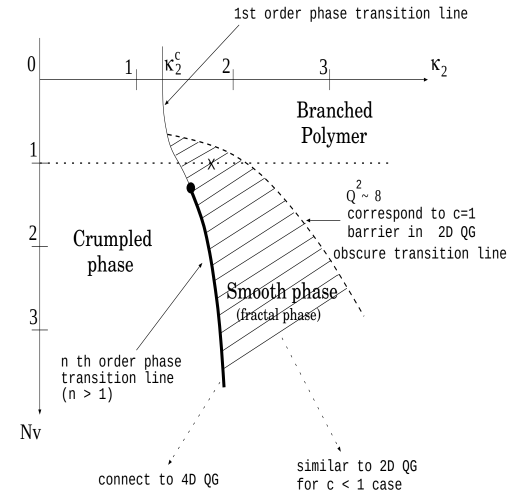

Figure 3. Rough sketch of the phase diagram. The $N_V = 1$ case has been intensively investigated, and we have obtained the scaling of the boundary volume distributions in Fig.2 at the point indicated by the cross.

has the possibility of taking a continuum limit. We have also investigated the boundary volume distribution in the crumpled and the branched polymer phases. In the crumpled phase we find that one mother universe is dominant. On the other hand, in the branched polymer phase we have no evidence for the existence of a mother universe. There is one further observation that we must not ignore in the region $\kappa_2 > \kappa_2^o$. The number of nodes of the manifolds is very close to its upper kinematic bound, $\frac{N_0}{N_4} \approx \frac{1}{4}$. This upper kinematic bound of the simplexes serves as evidence of a branched polymer. The phase transition at κ_2^c becomes softer the larger N_V becomes. Actually, in the $N_V = 3$ case a single peak in the node susceptibility is reported by the authors in ref.[4]. Unfortunately, even in the $N_V = 1$ case we have observed a discontinuity at the critical point κ_2^c, which is consistent with ref.[4].

4. Summary and Discussions

Let us summarize the main points made in the previous section. In Fig.3 we show a rough sketch of the phase diagram of 4D simplicial gravity. We have three phases in this parameter space: a crumpled phase, a smooth phase (shaded portion) and a branched polymer. The thin line denotes a discontinuous phase-transition line which is known in pure gravity; the a thick line denotes a smooth phase-transition line. In the smooth phase with $N_4 = 16K$ ($\kappa_2 = 1.7$) and $N_v = 1$ we obtained $\gamma_{st} = -0.38(5)$, $d_f = 2.8(5)$ and a good scaling relation of the boundary volume distributions with the scaling variable $x = V/D^{d_f - 1}$. The scaling structure of this smooth phase is similar to that of a 2D random (fractal) surface. It suggests the existence of a new smooth phase in 4D simplicial gravity. We obtained an obscure transition line (a broken line in Fig.3), and suggest that the obscure transition corresponds to the $c = 1$ barrier in 2D quantum gravity. The existence of genuine 4D quantum gravity on the critical point κ_2^c remains a matter for discussion.

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